Section A – Calculator Free – Time Allowed: 25 minutes

1. [1, 1, 2 = 4 marks]

   Given the vectors a, b and c shown in the diagram below, represent the resultant of:

   (a) $-a + 2b$ (b) $2c - b$
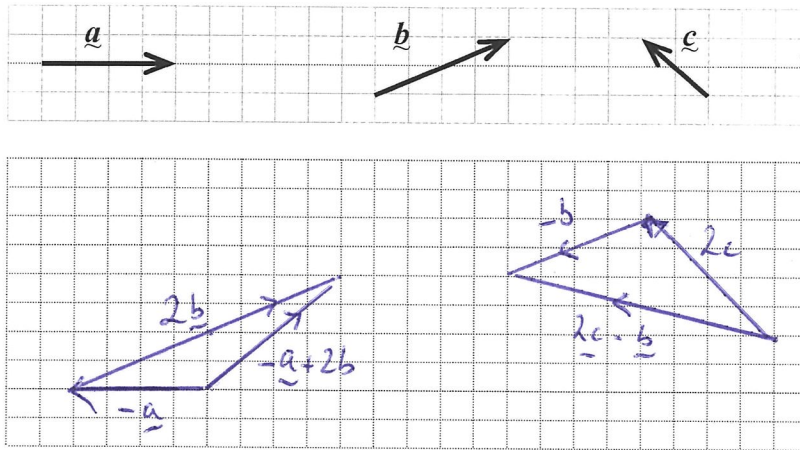
   No direction shown −1m

   (c) Express a in terms of b and c.

   $$b - c = 1.5a \quad \checkmark$$

   $$\therefore \quad a = \frac{2}{3}(b - c) \quad \checkmark$$

2. [3 marks]

   Express the following vector in the form $ai + bj$. Give a and b as exact rationalised values.

   $$\frac{\sqrt{3} \times 10}{2}$$

   $30°$ X

   $10N$ $-5N$
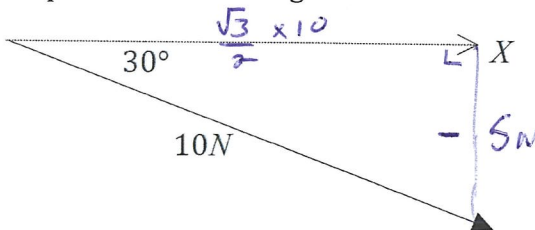
   $$\therefore \quad 5\sqrt{3}\,i - 5j \quad \checkmark$$

3.   [1, 2, 3, 2= 8 marks]

Given $a = -3i + 2j$ and $b = 5i - j$ determine exactly:

(a) $-2a + 3b$

$6i - 4j + 15i - 3j = 21i - 7j$

(b) $|a + b|$

$= |2i + j|$

$= \sqrt{5}$

(c) $|a| + |b|$ in the form $\sqrt{x}\left(\sqrt{y} + \sqrt{z}\right)$

$= \sqrt{13} + \sqrt{26}$

$= \sqrt{13}\left(\sqrt{1} + \sqrt{2}\right)$

(d)   A vector that is parallel but opposite to a with a magnitude of 5.

$\dfrac{-5\checkmark}{\sqrt{13}}\checkmark \left(-3i + 2j\right) = \dfrac{+15i - 10j}{\sqrt{13}}$

$= \dfrac{-5\sqrt{13}}{13}\left(-3i + 2j\right) \qquad = \dfrac{15\sqrt{13}\,i - 10\sqrt{13}\,j}{13}$

4. [2 marks]

The "SS Aardvark" is at position $\langle 20,10 \rangle$ at 2 p.m.
It now begins to move with a velocity vector of $7\underset{\sim}{i} - \underset{\sim}{j}$ km/h. If it continues with this velocity what will be its position at 1700?

3 hours $\therefore$ $41\underset{\sim}{i} + 7\underset{\sim}{j}$
$\underset{\sim}{c}$

5. [4 marks]

Vectors **a**, **b**, and **c** are such that $a = 3i + 4j, b = xi - 8j$ and $c = yi + 7j$. Given that **a** and **b** are parallel and **b** and **c** have equal magnitudes, find the values of x and y.

$b = \top (3i + 4j)$

$= \cancel{3} i + 8j$

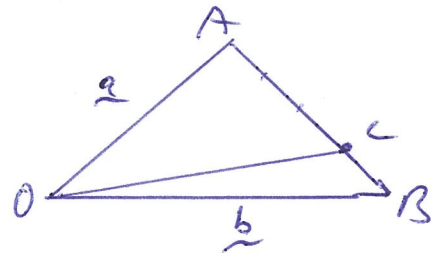$\therefore \top = -2$

$x = -6$

$|b| = 10$ ✓

$\Rightarrow 10^2 = y^2 + 7^2$ ✓

$y = \pm\sqrt{51}$ ✓

6. [1, 1, 2=4 marks]

OAB is a triangle with C a point on $\overline{AB}$ such that $\overrightarrow{AC} = \frac{3}{4}\overrightarrow{AB}$. If $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$, express in terms of **a** and or **b**:

(a) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

(b) $\overrightarrow{CB} = \frac{1}{4}(\underset{\sim}{b} - \underset{\sim}{a})$

(c) $\overrightarrow{OC} = \underset{\sim}{b} \cancel{+} \frac{1}{4} + \frac{1}{4}\underset{\sim}{a}$

$= \overrightarrow{OB} + \overrightarrow{BC} = \frac{3b}{4} + \frac{1}{4}\underset{\sim}{a}$
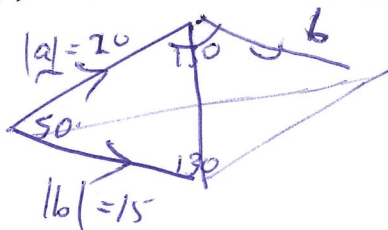
$\frac{\square}{10}$

Name: So/h.

Section B – Calculator and Notes Allowed – Time Allowed: 30 minutes

1. [1, 2, 2 = 5 marks]

   The angle between vectors **a** and **b** is 50°. Given that $|a| = 20$ metres and $|b| = 15$ metres:
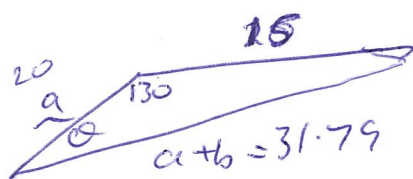   (a) make a sketch of the situation in the space provided.

   (b) Use the rules of trigonometry to find $|a + b|$.

   $$|a+b|^2 = 20^2 + 15^2 - 2 \times 20 \times 15 \cos 130°$$

   $$|a+b| = \sqrt{1010.67}$$

   $$= 31.79 \text{ m}$$

   (b) Find the size of the angle between $a + b$ and a.


   $$\frac{\sin \theta}{15} = \frac{\sin 130}{31.79}$$

   $$\theta = 21.189°$$

2.   [5 marks]

A student wishes to paddle her canoe West across the river from Point A on one bank to the jetty
which is on the opposite bank, directly opposite A. The student can paddle at a steady 5 km/h in
still water. However, today the river is flowing South at 2 km/h. If the river is 500 m wide at that
point, find the direction she should paddle and the time it will take to cross the river.
A clearly labelled diagram is for full marks.
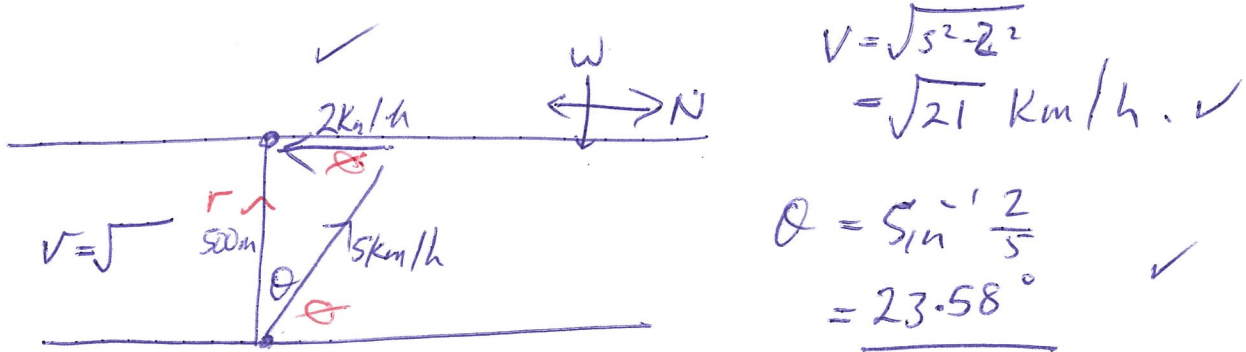
$$V = \sqrt{5^2 - 2^2}$$
$$= \sqrt{21} \text{ Km/h.} \checkmark$$

$$\theta = \sin^{-1} \frac{2}{5}$$
$$= 23.58°$$

$$\therefore \text{ Direction } 270° + 23.58°$$
$$= 293.58° T \checkmark$$

$$\text{Time } \frac{0.5}{\sqrt{21}} = 0.1091 \, h = 6 \min 33 \sec.$$

3.   [5 marks]

Using $a = 2i + 3j$ and $b = 4i - j$, express $6i - 4j$ in the form $\lambda \underset{\sim}{i} + \mu \underset{\sim}{j}$ leaving λ and μ as fractions.

$$\langle 6, -4 \rangle = 2T \underset{\sim}{i} + 3T \underset{\sim}{j} + 4M \underset{\sim}{i} - M \underset{\sim}{j} \quad \checkmark$$

$$\checkmark \therefore 6 = 2T + 4M$$
$$\checkmark \quad 4 = 3T - M$$
$$\Big\} \Rightarrow \begin{array}{l} 6 = 2T + 4M \\ -16 = 12T - 4M \end{array} \Rightarrow -10 = 14T$$

$$T = \frac{-5}{7}$$
$$M = \frac{13}{7}$$

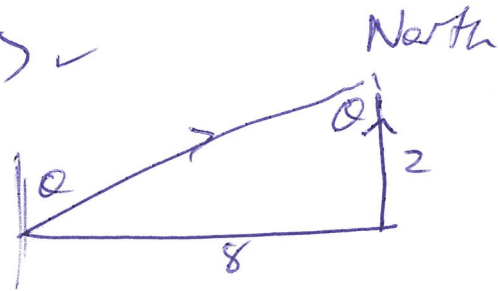$$\therefore \langle 6, -4 \rangle = \frac{-5}{7} \underset{\sim}{a} + \frac{13}{7} \underset{\sim}{b} \quad \checkmark$$

4. [4 marks]

F_1, F_2 and F_3 are all forces that act simultaneously on a body. F_1 is measured at $\binom{2}{3}$ N, F_2 at $\langle 4,3 \rangle$ N and $F_3 = 2i-4j$ N. Find the exact magnitude of the resultant force acting on the body and it's direction correct to the third decimal place. <u>Use North as the direction of j.</u>
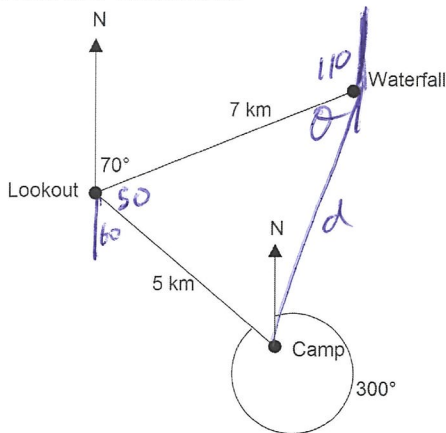
$r = F_1 + F_2 + F_3$

$= \langle 2+4+2, 3+3, -4 \rangle$ ✓

$= \langle 8, 2 \rangle$ ✓

∴ $\underline{r}$ is $\sqrt{68}$ ✓ at $\tan^{-1}(4) = 075.964°$ ✓

5. [4 marks]

Carefully study the diagram that shows a journey taken by a hiker from camp to a waterfall, via a scenic lookout. Find the direct distance of the waterfall from camp **and** the bearing of the camp from the waterfall.

$d = \sqrt{7^2+5^2-2\times7\times5\cos50}$ ✓

$= 5.3856167$ km ✓

$\theta = \sin^{-1}\left(\dfrac{5\sin50}{5.3856167}\right)$

$= 45.3322°$ ✓

∴ Direction $360 - 110 - 45.32$

$= 204.66°T$.

6. [3 marks]
Arnie has parked his car and trailer on a hill which is sloping at 15° to the horizontal. He intends to unhitch the trailer from the car and push the trailer up the hill himself.
The trailer exerts a force due to gravity of 2000N vertically and Arnie can push with a force of 700N parallel to the hill. Will he be able to move the trailer? Justify your answer.

$F = 2000 \sin 15° = 517N$ ✓

$2000 \sin 15$

$\therefore$ yes as $700N > 577N$ ✓

$2000N$
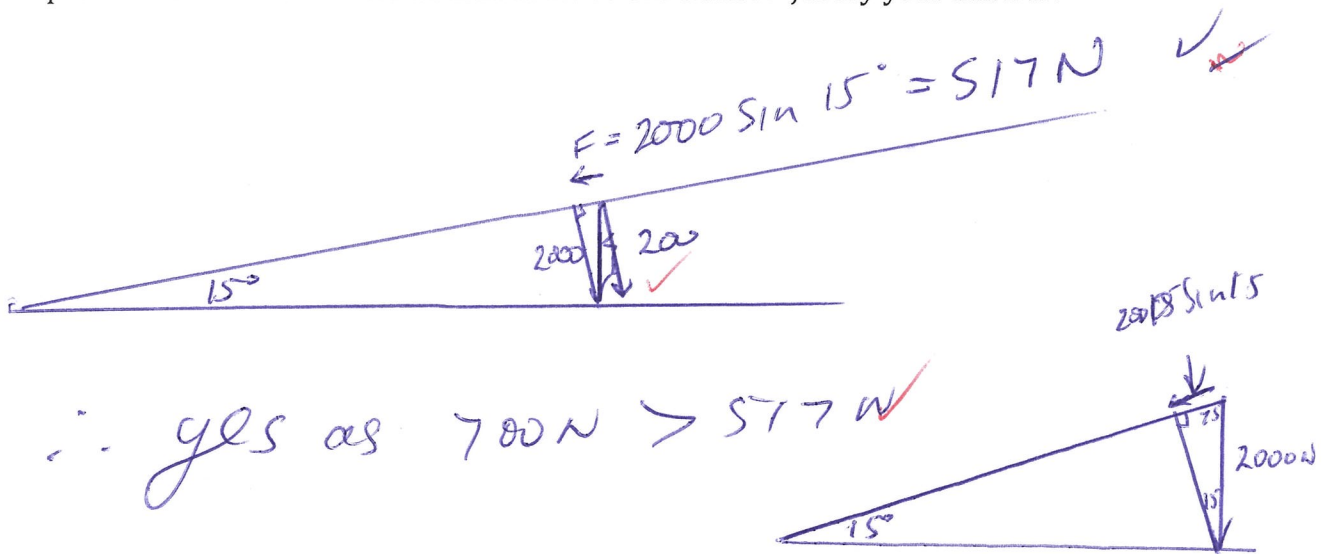
7. [3 marks]

Janine was travelling due north at 60 km/h and turned (taking 5 seconds) so that she is now travelling due East at 80 km/h. Find the direction and magnitude of her acceleration.

$a = \dfrac{V - u}{t}$

$5 \text{ Seconds} = \frac{1}{720} \text{th Hour}.$

80Km/h

$5 \underset{\sim}{a}$
$= 100 \text{Km/h}$
$=$

60Km/h ✓
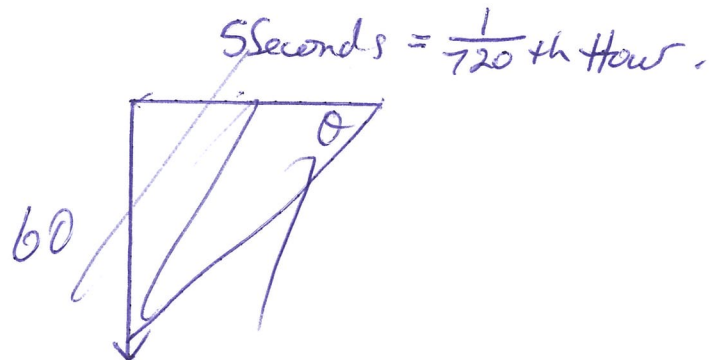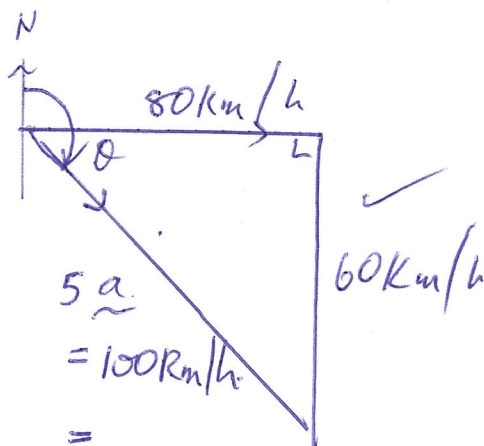
5 sec need △ Speed in Sec.
OR 5 Sec as hours

$\dfrac{100}{\frac{1}{720}} = \underline{72000 \text{Km/h}^2}$

at $90 + \tan^{-1}\left(\frac{3}{4}\right)$

$= \underline{126.9°}$ ✓

OR $100 \text{km/h} = \dfrac{100\,000\,m}{\frac{3600\,s}{5}} = 5.5\, m/s^2.$